

## CS 131 – Fall 2019, Prof. Tsourakakis

Assignment 3 must be submitted by Friday, September 27, on Gradescope.

**Problem 1.** Let predicate  $C(x, y)$  mean that student  $x$  is enrolled in class  $y$ , where the domain for  $x$  consists of all students in BU, and the domain for  $y$  consists of all classes being given at BU. Express each of the following statements by a simple English sentence.

- a) (2 pt)  $C(\text{Randy Goldberg}, \text{CS131})$
- b) (3 pt)  $\exists x C(x, \text{CS111})$
- c) (5 pt)  $\exists y C(\text{Carol Sitea}, y)$
- d) (5 pt)  $\exists x (C(x, \text{CS111}) \wedge C(x, \text{CS131}))$
- e) (5 pt)  $\exists x \exists y \forall z ((x \neq y) \wedge (C(x, z) \rightarrow C(y, z)))$
- f) (5 pt)  $\exists x \exists y \forall z ((x \neq y) \wedge (C(x, z) \leftrightarrow C(y, z)))$

**Problem 2.** Argue whether each of the following arguments is valid or not. For the valid arguments, which rule of inference is used? For the invalid arguments, explain why they are invalid.

- a) (5 pt) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
- b) (5 pt) It is either hotter than 100 degrees today or pollution is dangerous. It is less than 100 degrees today. Therefore, pollution is dangerous.
- c) (5 pt) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
- d) (5 pt) Steve will work in the computer company this summer. Therefore, this summer Steve will work at a computer company or he will be beach bum.
- e) (5 pt) If I exercise every day, I will become an athlete. I am an athlete. Therefore, I exercise every day.
- f) (5 pt) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

**Problem 3.** Prove the following statements:

- a) (10 pt) Let  $n$  be an arbitrary integer. If  $3n + 2$  is even, then  $n$  is even.
  - i) Identify what is given and what is asked in this problem.
  - ii) Outline what is given and asked in a direct proof, a proof by contraposition and in a proof by contradiction.
  - iii) Prove the statement by contraposition and contradiction.
- b) (10 pt) Let  $n, m$  be arbitrary integers. If  $mn$  is even, then  $n$  is even or  $m$  is even.

iv) Identify what is given and what is asked in this problem.

v) Outline what is given and asked in a direct proof, a proof by contraposition and in a proof by contradiction.

vi) Prove the statement by contraposition and contradiction.

**Problem 4.** Let  $a, b$  be arbitrary integers. Prove if  $a$  is multiple of 3, and  $b$  is even, then  $ab$  is a multiple of 6.

a) (5 pt) Express the statement and its negation in predicate logic.

b) (10 pt) Explain as in class, what is given to you (logical premises), and the goal (conclusion).

c) (10 pt) Outline a proof strategy (e.g., direct, contrapositive, contradiction) that you can use to prove the theorem. In your outline, you should explain what the logical premises are and the conclusion. Use your proof strategy to prove the statement.