CS 131 – Fall 2019, Prof. Tsourakakis
Assignment 5 must be submitted by Friday October 11, 2019 5:00pm, on Gradescope.

Problem 1. (18 points) Express the following sums and products by using $\sum$ and/or $\prod$ notations:

a) (2 points) $1 + 8 + 27 + 64 + 125 + ...$

b) (2 points) $2 + 5 + 8 + 11 + ... + 29$

c) (2 points) $-8 + 11 - 14 + 17 - 20$

d) (2 points) $3 - 6 + 12 - 24 + ...$

e) (2 points) $\frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \frac{1}{9}$

f) (2 points) $1 \cdot \frac{1}{4} \cdot \frac{1}{16} \cdot \frac{1}{64} \cdot \frac{1}{256}$

g) (2 points) $n!$

h) (4 points) $24 + 384 + 1944 + 6144 + 15000$. Express this series in the form $\sum_{i=1}^{5} \prod_{j=1}^{4} f(i, j)$ where $f(i, j)$ is an appropriate function of summation indices i, j.

Problem 2. (17 points) Evaluate/Compute the following sums and product sums:

a) (4 points) $\sum_{i=1}^{10} \sum_{j=1}^{5} j^2 i$

b) (4 points) $\prod_{i=1}^{3} \frac{1}{i^{2i}}$

c) (4 points) $\sum_{i=1}^{3} \prod_{j=1}^{3} \frac{i}{j}$

d) (5 points) $\prod_{i=1}^{3} \prod_{j=1}^{3} \frac{j+i}{2j}$

Problem 3. (25 points) Let $f_0, f_1, f_2, \ldots$ be the Fibonacci sequence defined as: $f_0 = 0, f_1 = 1$, and for every $k > 1$, $f_k = f_{k-1} + f_{k-2}$. This problem will help us understand how quickly this sequence grows. In each of the following parts, you can use previous parts, even if you haven't solved them.

a) (10 points) Use induction to prove that for all $n \geq 0$, $f_0 + f_1 + \cdots + f_n = f_{n+2} - 1$.

b) (8 points) Use induction to prove that for every $n \geq 2$, $f_n \geq (1.5)^{n-2}$. Note: you'll have two base cases, f_2 and f_3. For the inductive case of f_{k+1}, you'll need to use the inductive hypothesis for both k and $k-1$. This method of doing induction is still fine, because we can define a predicate $P(k)$ to be $(f_{k-1} \geq 1.5^{k-3} \wedge f_k \geq 1.5^{k-2})$ and prove it's true for all $k \geq 2$ with the usual induction.

c) (7 points) Use induction to prove that for every $n \geq 0$, $f_n \leq 2^{n-1}$.

Problem 4. (20 points)

a) (10 points) Use induction to prove $\forall n \in \mathbb{Z}^+ \ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

b) (10 points) Use induction to prove $\forall n \in \mathbb{Z}^+ \; \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$.

Problem 5. (20 points)

a) (10 points) Assume that X, Y, and Z are sets. Prove $X \setminus (Y \setminus Z) \subseteq (X \setminus Y) \cup Z$.

b) (10 points) Let A be an arbitrary set. Prove by using induction that the cardinality of the power set of A is equal to $2^{|A|}$.