CS 131 – Fall 2019, Prof. Tsourakakis

Assignment 6 must be submitted by Friday October 25, 2019 5:00pm, on Gradescope.

Problem 1. (36 pts). Alice wants to prove by induction that a predicate P holds for certain nonnegative integers. She has proven that for all nonnegative integers $n = 0, 1, \ldots$

$$P(n) \rightarrow P(n+3).$$

(a) (2 points each) Which of the following, if also proven true, would enable Alice to conclude $\forall n \geq 5.P(n)$? Just answer yes or no.

1. $P(5)$ and $P(6)$
2. $P(0), P(1)$, and $P(2)$
3. $P(2), P(4)$, and $P(5)$
4. $P(3), P(5)$, and $P(7)$

(b) (2 points each) Suppose Alice manages to also prove that $P(5)$ holds. Which of the following propositions can she now deduce? Answer yes or no with a one sentence explanation, separately for each part.

1. $P(n)$ holds for all $n \geq 5$.
2. $P(3n)$ holds for all $n \geq 5$.
3. $P(n)$ holds for $n = 8, 11, 14, \ldots$
4. $P(n)$ does not hold for any $n < 5$.
5. $P(3n + 5)$ holds for all $n \geq 0$.
6. $P(3n - 1)$ holds for all $n > 2$.
7. $P(0) \rightarrow (\forall n \geq 0.P(3n + 2))$.
8. $P(0) \rightarrow (\forall n \geq 0.P(3n))$.

Problem 2. (30 points)

In this problem you will be given a few functions and will be asked to do computations related to them.

a) (10 points) Let function $F(n, m)$ outputs n if $m = 0$ and $F(n, m - 1) + 1$ otherwise.

1. Evaluate $F(10, 6)$.
2. Write a recursion of the running time and solve it.
3. What does $F(n, m)$ compute? Express it in terms of n and m.

b) (10 points) Let function $F(n, m)$ outputs 1 if $m = 0$ and $F(n, m - 1) * n$ otherwise.

1. Evaluate $F(2, 7)$.
2. Write a recursion of the running time and solve it.
3. What does $F(n, m)$ compute? Express it in terms of n and m.

c) (10 points) Let function $F(n, m)$ outputs 1 if $m = 0$; if $m \neq 0$ and m is even, $F(n, m) = F(n, m/2)^2$, if $m \neq 0$ and m is odd, $F(n, m) = F(n, m-1) * n$.

1. Evaluate $F(2, 7)$.
2. Write a recursion of the running time and solve it.
3. What does $F(n, m)$ compute? Express it in terms of n and m.

Problem 3. (20 points)

Every road in country X is one-way. Every pair of cities is connected by exactly one direct road (going in only one direction). Show that there exists a city which can be reached from every other city either directly or via a route that goes through at most one other city. (Hint: Use induction on the number of the cities.)

Problem 4. (14 points)

For which values of n can a group of n people be divided into teams, where each team consists of exactly 4 or exactly 7 people? Use induction to prove your answer correct.