CS 131 – Fall 2019, Prof. Tsourakakis

Assignment 8 must be submitted by Friday November 15, 2019 5:00pm, on Gradescope.

Problem 1. (20 points) Prove or disprove the following statements:

a) (3 points) $\forall x \in \mathbb{R}, \lfloor x^2 \rfloor = (\lfloor x \rfloor)^2$

b) (3 points) $\forall x \in \mathbb{R}, \lfloor 2x \rfloor = 2\lfloor x \rfloor$

c) (6 points) $\forall x \in \mathbb{R}, \lceil \lfloor x \rfloor \rceil = \lfloor x \rfloor$

d) (8 points) $\forall x \in \mathbb{R}, \lceil -x \rceil = -\lfloor x \rfloor$

Problem 2. (15 points)

a) (5 points) Convert 101 from decimal to binary.

b) (5 points) Convert 206 from octal to binary.

c) (5 points) Convert CAF from hexadecimal to decimal.

Problem 3. (15 points) Prove the following statements without using induction:

a) (7 points) $\forall x \in \mathbb{Z}, 6|(x-1)x(x+1)$

b) (8 points) $\forall x \in \mathbb{Z}, 120|(x-2)(x-1)x(x+1)(x+2)$

Problem 4. (25 points) Prove that there are infinitely many primes with remainder 3 when divided by 4.

Problem 5. (25 points)

a) (8 points) Find Bezout's coefficients for $x \cdot 122 + y \cdot 16 = gcd(122, 16)$

b) (17 points) Definition: The **least common multiple** (lcm) of the positive integers a and b is the smallest positive integer that is divisible by both a and b. The least common multiple of a and b is denoted by $lcm(a, b)$. For example, $lcm(4, 6) = 12$. Note that the least common multiple always exists.
Prove $gcd(a, b) \cdot lcm(a, b) = ab$ by using two different methods that are listed in each part:

i) (9 points) Use prime factorizations to a and b.

ii) (8 points) Use the theorem in slide 227. The theorem is: The gcd of a and b is equal to the smallest positive linear combination of a and b. Note that "smallest positive linear combination" means smallest positive number which is a linear combination.