

CS 131 – Fall 2019, Prof. Tsourakakis

Assignment 9 must be submitted by Friday November 22, 2019 5:00pm, on Gradescope.

Problem 1. (10 points) Calculate a value of Euler function φ as a function of n :

- a) (5 points) $\varphi(7^n)$
- b) (5 points) $\varphi(5^n 2^{n+3})$

Problem 2. (10 points) Prove the following statements:

- a) (5 points) Prove that for positive integer n , $n^3 + 2n$ is divisible by 3.
- b) (5 points) Prove that for positive integer n , $n^3 - n$ is divisible by 6.

Problem 3. (20 points + 10 additional points) Solve the following equations in non-negative integers.

- a) (5 points) $x^2 - y^2 = 221$
- b) (5 points) $a + b = ab$
- c) (10 points) $\gcd(a, b) \operatorname{lcm}(a, b) = b + 9$
- d) (10 additional points) $x^4 + 2x^3 - y^2(1 + 2x) + x^2(1 - y^2) = 2299$

Problem 4. (35 points) How many positive integers less than 1000

- a) (5 points) are divisible by 7 but not by 11?
- b) (5 points) are divisible by both 7 and 11?
- c) (5 points) are divisible by either 7 or 11
- d) (5 points) are divisible by exactly one of 7 and 11?
- e) (5 points) are divisible by neither 7 nor 11?
- f) (5 points) have distinct digits?
- g) (5 points) have distinct digits and are even?

Problem 5. (25 points)

- a) (5 points) Find multiplicative inverse (or prove that it doesn't exist) of 3 mod 11 using remainders table.
- b) (5 points) Find multiplicative inverse (or prove that it doesn't exist) of 6 mod 11 using remainders table.
- c) (5 points) Find multiplicative inverse (or prove that it doesn't exist) of 2 mod 12 using remainders table.
- d) (10 points) Find a multiplicative inverse of 247 mod 154 using Bezout's coefficients.